Supplementary material for: Small-group monopolistic competition in a global computable general equilibrium model: Meeting the Markusen challenge

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A.1. Introduction

The aim of this supplementary material is to provide the technical details underlying the theory and implementation of an MM specification in a standard GTAP model.

We start in section A.2 by describing a set of encompassing equations. These equations identify all Widget varieties, not just typical varieties. Following Melitz, we show how the variety dimension can be reduced to typical varieties.

Section A.3 is a description and solution of the optimization problem that leads to the specification in (T1.7e) in Table 1 of the main paper of the modifying factor (Z_{sd}) in the SGMC determination of the minimum-productivity variety on the sd-link.

In section A.4, we derive percentage change versions of the equations for an MM industry. The coefficients in the percentage-change equations are readily interpretable in terms of cost shares, sales shares, substitution elasticities and parameters of the Pareto distribution of variety productivities. The percentage change equations, with their relatively easily evaluated coefficients are the basis for implementing MM in GEMPACK software.

Section A.5 describes our method for creating an MM industry in the GTAP model with minimal changes to standard GTAP. The method relies on: technical change variables to capture economies of scale implied by fixed costs; tax variables to represent profits and to capture differences across s-to-d trade links in prices charged by the Widget industry in country s; and preference variables to capture love-of-variety.

The final section, A.6, strengthens the interpretation of the simulation described in section 3 of the main paper. It explains how a reduction in competition can generate an increase in varieties and an increase in the costs to consumers of MM products.

A.2. Deriving MM equations from encompassing equations

The first column of Table A.1 repeats the first 6 groups of MM equations from Table 1 in section 2 of the main paper. These are either straight Melitz or slightly adapted from Melitz. The second column is an encompassing or more general specification of a Widget industry. Notational notes on the encompassing equations are at the end of the Table A.1. Notation for the MM equations was given at the end of Table 1 of the main paper.

In this section we derive the MM equations in the first column of Table A.1 as special cases of the encompassing equations in the second column.

Commentary on the encompassing equations (second column in Table A.1)

Encompassing equation (T1.1a) determines the prices of the varieties on the sd-link by marking up their marginal costs. We could give the markup factor k and s subscripts in addition to the d subscript. However, we don't have any implementable theory to suggest differences in markups across varieties in d's market.

Table A.1. MM and Encompassing equations for a Widget industry (notation next page)

	Equations for Melitz-Markusen (MM)	Encompassing equations
(T1.1a)	$P_{\bullet sd} = \left(\frac{W_s T_{sd}}{\Phi_{\bullet sd}}\right) * M_d$	$P_{ksd} = \left(\frac{W_s T_{sd}}{\Phi_k}\right) * M_d \qquad k \in S(s,d)$
(T1.1b)	$\Phi_{\bullet sd} = \beta * \Phi_{\min(s,d)}$	
(T1.2a)	$P_d = \left(\sum_s N_{sd} \delta_{sd}^{\sigma} P_{\bullet sd}^{1-\sigma}\right)^{\frac{1}{(1-\sigma)}}$	$P_d = \left(\sum_s \sum_{k \in S(s,d)} N_s B_s g_s(\Phi_k) \delta_{sd}^{\sigma} P_{ksd}^{1-\sigma}\right)^{1/(1-\sigma)}$
(T1.2b)	$N_{sd} = N_s * B_s * \left(\Phi_{\min(s,d)}\right)^{-\alpha}$	
(T1.3a)	$Q_{\bullet sd} = Q_d \delta_{sd}^{\sigma} \left(\frac{P_d}{P_{\bullet sd}}\right)^{\sigma}$	$Q_{ksd} = Q_d \delta_{sd}^{\sigma} \left(\frac{P_d}{P_{ksd}}\right)^{\sigma}$
(T1.3b)	$V_{sd} = P_{\bullet sd} N_{sd} Q_{\bullet sd}$	$V_{sd} = \sum_{k \in S(s,d)} P_{ksd} N_s B_s g_s(\Phi_k) Q_{ksd}$
(T1.3c)	$Q_{sd} = N_{sd} Q_{\bullet sd}$	
(T1.4a)	$(M_d - 1)\left(\frac{Q_{\min(s,d)}}{\Phi_{\min(s,d)}}\right) - F_{sd} * Z_{sd} = 0$	$\mathrm{Min}_{k \in S(s,d)}\left[\Pi_{ksd}\right] - W_s F_{sd}(Z_{sd} - 1) = 0$
(T1.4b)	$Q_{\min(s,d)} = Q_{\bullet sd} / \beta^{\sigma}$	$\Pi_{ksd} = \frac{P_{ksd}}{T_{sd}} Q_{ksd} - \left(\frac{W_s}{\Phi_k}\right) Q_{ksd} - F_{sd} W_s$
(T1.5)	$\Pi tot_s = \sum_d N_{sd}\left[\frac{W_s}{\Phi_{\bullet sd}} * Q_{\bullet sd} * (M_d - 1) - W_s * F_{sd}\right] - N_s H_s W_s$	$\Pi tot_s = \sum_d \sum_{k \in S(s,d)} N_s B_s g_s(\Phi_k) \Pi_{ksd} - N_s H_s W_s$
(T1.6)	$L_s = \sum_d \frac{N_{sd} Q_{\bullet sd}}{\Phi_{\bullet sd}} + \sum_d N_{sd} F_{sd} + N_s H_s$	$L_s = \sum_d \sum_{k \in S(s,d)} N_s B_s g_s(\Phi_k) \frac{Q_{ksd}}{\Phi_k} + \sum_d \sum_{k \in S(s,d)} N_s B_s g_s(\Phi_k) F_{sd} + N_s H_s$

(Continued…)

Notation for the encompassing equations [Notation for the MM equations is at the end of Table 2.1.]

N_s is the number of Widget-producing firms in country s.

S(s,d) is the set of all varieties k produced in s that are sent from s to d. With all varieties in s facing the same sd-link set up cost, we can assume that if any class-k variety is sent on the sd-link, then all varieties in s with productivity greater than or equal to Φ_k are sent on the sd-link.

$g_s(\Phi_k)$, assumed the same for all firms in s, is the proportion of varieties producible by an s firm that have productivity level Φ_k. This is the number of additional units of output generated per additional unit of input. In this theoretical exposition, we assume that the only input is labor but in our implementation of the theory in GTAP, Widget producers use a bundle of inputs that include labor, capital, land and intermediates. When we refer to varieties in class k in country s, we mean the set of varieties in s that have productivity Φ_k.

B_s is the number of varieties potentially producible by a firm in s. Thus, the number of k-class varieties potentially producible in country s is $N_s B_s g_s(\Phi_k)$.

P_{ksd} is the price in country d of class-k Widgets produced in country s. We assume that all class-k Widgets sent on the sd-link have the same price.

W_s is the cost of a unit of input (labor) to Widget makers in country s.

T_{sd} is the power (1 plus rate) of the tariff or possibly transport costs associated with the sale of Widgets from s to d.

M_d is the markup on marginal costs (≥1) applied on all varieties sent to d.

F_{sd} is the fixed cost (measured in units of input) required to set up sales of a variety from s to d. We refer to this as the sd-link set up cost.

H_s is the fixed cost (measured in units of input) to set up a firm in s.

δ_{sd} is a positive parameter reflecting d's preference for varieties in general from s relative to those from other countries.

σ (restricted to be >1) is the elasticity of substitution between varieties, assumed to be the same for all consumers in every country and for any pair of varieties wherever sourced.

Q_{ksd} is the quantity of Widgets sent from s to d of each variety in class k (this includes the s-to-s flows).

Q_d is the total requirement for Widgets in d. Q_{ksd} is a normal quantity (count or tonnes). Q_d is a CES aggregate of the Q_{ksd} s.

P_d is the cost in region d of satisfying a unit demand for Widgets, that is the cost of increasing Q_d by 1.

Π_{ksd} is the contribution to the profits of a producer in s from its sales to d of k-class Widgets.

Z_{sd} is a variable with value greater or equal to 1. As explained in section 2 in the discussion of (T1.4a), Z_{sd} takes account of the assessment of producers in s of the effect on their profits from established varieties on the sd-link of adding an extra variety. If their assessment is that there is no effect, then Z_{sd} is 1.

Πtot_s is total profits for Widget producers in s

L_s is the total number of input bundles (employment) used in the Widget industry in country s.

Consequently, even in the encompassing equations we assume that the markup factor (M_d) is the same for all varieties used in d.

Encompassing equation (T1.3a) determines the demand in country d for each k-class variety from s. By k-class varieties, we mean varieties with marginal productivity of Φ_k. Encompassing equation (T1.2a) determines the cost to Widget users in s of an extra unit of Widgets (a unit increase in Q_d). These two equations follow from a CES cost-minimizing problem of the form:

choose Q_{ksd} for all s and $k \in S(s,d)$

$$\text{to minimize } \sum_{s} \sum_{k \in S(s,d)} P_{ksd} Q_{ksd} N_s B_s g(\Phi_k) \tag{A.2.1}$$

subject to

$$Q_d = \left(\sum_{s} \sum_{k \in S(s,d)} \delta_{sd} N_s B_s g(\Phi_k) Q_{ksd}^{(\sigma-1)/\sigma} \right)^{\sigma/(\sigma-1)} \tag{A.2.2}$$

In interpreting (A.2.1) and (A.2.2) it is useful to note that $N_s B_s g_s(\Phi_k)$ is the number of k-class varieties sent on the sd-link, that is the number of firms *times* the number of potentially producible varieties per firm *times* the proportion of varieties with productivity Φ_k.

Encompassing equation (T1.3b) is an accounting relationship determining the value of the sd-flow.

Encompassing equation (T1.4b) defines profits for a k-class variety in s from its sales to d as: revenue (net of tariffs and transport costs) *less* variable costs of production *less* the fixed costs required to set up sales of a variety on the sd-link. Then via encompassing equation (T1.4a) we assume that for a variety to be sent on the sd-link, sending it on that link must contribute non-negatively to the profits of Widget-producers in s. As explained in section 2 of the main paper in the discussion of MM equation (T1.4a), the Z_{sd} factor, which is greater than or equal to 1, takes account of the perceived effect on the profitability of established varieties on the sd-link of an extra variety. In the encompassing equations, we leave the determination of Z_{sd} open.

Encompassing equation (T1.5) defines total profits in the Widget industry of s as the sum of profit contributions over all classes and links *less* the fixed costs of setting up Widget firms in s.

Encompassing equation (T1.6) defines total input bundles (employment) in the Widget industry in s as the sum of bundles used as variable inputs and fixed inputs.

Introducing the Pareto distribution and defining the typical variety

In most Melitz-based CGE models, it is assumed that the productivity distribution from which intending Widget entrepreneurs make their productivity draw has the Pareto form:

$$g_s(\Phi) = \alpha \Phi^{-\alpha-1}, \ \Phi \geq 1 \tag{A.2.3}$$

where α is a positive parameter. Under (A.2.3), the lowest potential productivity value is 1. This assumption can be made without loss of generality through a suitable choice of units for the input bundle.

We adopt (A.2.3) to describe the distribution of productivities across varieties producible by a firm in s, where $g_s(\Phi)$ is, as defined earlier, the proportion of the firm's varieties that has productivity Φ.

Next, following Melitz, we define the typical variety sent on the sd-link as one that has productivity level given by

$$\Phi_{\bullet sd} = \left[\sum_{k \in S(s,d)} \frac{N_s B_s g_s(\Phi_k)}{N_{sd}} \Phi_k^{\ \sigma-1} \right]^{\frac{1}{\sigma-1}} \quad , \tag{A.2.4}$$

that is, the typical variety on the sd-link is a CES average of the productivities of all the varieties k on the sd-link, $k \in S(s,d)$. As shown by Dixon *et al.* (2018, chapter 2), the definition in (A.2.4) can be reduced to a statement about the "size" of the typical variety. It turns out that the typical variety on the sd-link is one which uses the average number of input bundles over varieties on the sd-link to generate output for the link. For example, if production for the sd-link (not link or firm set-up costs) uses 300 input bundles and there are 15 varieties sent on the sd-link, then the typical variety on the link is one whose production for the link requires 20 input bundles.

With the definition of the typical variety for the sd-link in place, we can move to the derivation of the MM equations in the first column of Table A.1.

Deriving MM equation (T1.1a)

MM equation (T1.1a) is the encompassing equations written for the typical variety on the sd-link.

Deriving MM equation (T1.2b)

From (A.2.3) we obtain

$$\int_{\Phi_{min}}^{\infty} g_s(\Phi)\, d\Phi = \Phi_{min}^{-\alpha} \tag{A.2.5}$$

(A.2.5) means that the proportion of productivity values in country s that are greater than any given level, Φ_{min}, is $\Phi_{min}^{-\alpha}$. Thus the proportion of varieties in country s with productivity of at least $\Phi_{min(s,d)}$, i.e. the proportion of varieties $[N_{sd}/(N_s B_s)]$ that can be sent on the sd-link is $\Phi_{min(s,d)}^{-\alpha}$. This justifies the MM equation (T1.2b).

Deriving MM equation (T1.1b)

Next, we apply (A.2.3) and (T1.2b) in a continuous version of (A.2.4). This gives

$$\Phi_{\bullet sd}^{\sigma-1} = \int_{\Phi_{min(s,d)}}^{\infty} \Phi_{min(s,d)}^{\alpha} \alpha\, \Phi^{-\alpha-1} \Phi^{\sigma-1} d\Phi \tag{A.2.6}$$

that is

$$\Phi_{\bullet sd}^{\sigma-1} = \left(\frac{\alpha}{\alpha-(\sigma-1)} \right) \Phi_{min(s,d)}^{\sigma-1} \quad . \tag{A.2.7}$$

In deriving (A.2.7), we assume that

$$\alpha > (\sigma\text{-}1) \tag{A.2.8}$$

This doesn't have an obvious economic interpretation. However, without it, the integral on the RHS of (A.2.6) is unbounded. From (A.2.7) we get

$$\Phi_{\bullet sd} = \beta \Phi_{min(s,d)} \quad . \tag{A.2.9}$$

where

$$\beta = \left(\frac{\alpha}{\alpha - (\sigma - 1)} \right)^{1/(\sigma-1)} \quad . \tag{A.2.10}$$

This justifies (T1.1b).

Deriving MM equation (T1.2a)

Encompassing equation (T1.1a) implies that

$$P_{ksd} = P_{\bullet sd} * \frac{\Phi_{\bullet sd}}{\Phi_{ksd}} \tag{A.2.11}$$

Substituting from (A.2.11) into the encompassing version of (T1.2a) and using (A.2.3), we obtain:

$$P_d^{\,1-\sigma} = \sum_s N_s B_s \delta_{sd}^{\ \sigma} (P_{\bullet sd} \Phi_{\bullet sd})^{1-\sigma} \int_{\Phi \min(s,d)}^{\infty} \alpha \Phi^{-\alpha-1} \Phi^{\sigma-1} d\Phi \tag{A.2.12}$$

leading to

$$P_d^{\,1-\sigma} = \sum_s N_s B_s \delta_{sd}^{\ \sigma} (P_{\bullet sd} \Phi_{\bullet sd})^{1-\sigma} \frac{\alpha}{\alpha - (\sigma - 1)} * \Phi_{\min(s.d)}^{\sigma-\alpha-1} \tag{A.2.13}$$

Via MM equations (T1.1b) and (T1.2b) together with (A.2.10), we can reduce (A.2.13) to

$$P_d^{\,1-\sigma} = \sum_s N_{sd} \delta_{sd}^{\ \sigma} (P_{\bullet sd})^{1-\sigma} \tag{A.2.14}$$

which establishes MM equation (T1.2a).

Deriving MM equation (T1.3a)

This is the encompassing equation written for the typical variety on the sd-link.

Deriving MM equations (T1.3b) and (T1.3c)

From encompassing equations (T1.3a) and (T1.1a), we have

$$Q_{ksd} = Q_{\bullet sd} * \left(\frac{P_{ksd}}{P_{\bullet sd}} \right)^{-\sigma} \tag{A.2.15}$$

and

$$P_{ksd} = P_{\bullet sd} * \left(\frac{\Phi_{\bullet sd}}{\Phi_{ksd}} \right) \tag{A.2.16}$$

Then substituting into the continuous version of encompassing equation (T1.3b) and using MM equation (T1.2b) and (A.2.3), we obtain

$$V_{sd} = \int_{\Phi_{\min(s,d)}}^{\infty} P_{\bullet sd} * \left(\frac{\Phi_{\bullet sd}}{\Phi} \right) * N_{sd} * \left(\Phi_{\min(s,d)} \right)^{\alpha} \alpha \Phi^{-\alpha-1} * Q_{\bullet sd} * \left(\frac{\Phi_{\bullet sd}}{\Phi} \right)^{-\sigma} d\Phi \tag{A.2.17}$$

Rearranging (A.2.17) and using (T1.1b) to eliminate $\Phi_{\bullet sd}$ gives

$$V_{sd} = P_{\bullet sd} * N_{sd} * Q_{\bullet sd} * \beta^{1-\sigma} * \left(\Phi_{\min(s,d)} \right)^{1+\alpha-\sigma} \alpha \int_{\Phi_{\min(s,d)}}^{\infty} \Phi^{-\alpha-1+\sigma-1} d\Phi \tag{A.2.18}$$

By performing the integration and noting the definition of β in (A.2.10), we can reduce (A.2.18) to (T1.3b).

MM equation (T1.3c) defines the quantity (Q_{sd}) of Widgets on the sd-link as the value (V_{sd}) divided by the price of the typical variety ($P_{\bullet sd}$). Whereas V_{sd} feeds into other parts of the general equilibrium model, Q_{sd} does not. It is simply a convenient variable in reporting results. Consequently, we do not need to derive MM equation (T1.3c) from an encompassing version, or even to specify an encompassing version.

Deriving MM equations (T1.4a) and (T1.4b)

Encompassing equation (T1.4b) calculates the contribution to the profits of Widget producers in s from sending a k-class variety on the sd-link as:

revenue net of transport costs and tariffs,

less production costs,

less set-up costs for a variety on the sd-link.

By substituting from encompassing equation (T1.1a) into encompassing equation (T1.4b) we find that

$$\Pi_{ksd} = \left[M_d - 1\right] * \frac{W_s}{\Phi_k} * Q_{ksd} - F_{sd} W_s \tag{A.2.19}$$

Using encompassing equations (T1.3a) and (T1.1a) we can write (A.2.19) as

$$\Pi_{ksd} = \Phi_k^{\sigma-1} * \left(M_d - 1\right) * W_s * Q_d \delta_{sd}^{\sigma} P_d^{\sigma} \left(W_s T_{sd} M_d\right)^{-\sigma} - F_{sd} W_s \tag{A.2.20}$$

Recall that M_d and σ are greater than 1. Hence, (A.2.20) shows that Π_{ksd} is as an increasing function of Φ_k. This means that a variety with minimum profitability over all those sent on the sd-link also has the minimum productivity level, $\Phi_{min(s,d)}$. Now using (A.2.19), we see that encompassing equation (T1.4a) implies that

$$\left(M_d - 1\right) W_s \left(\frac{Q_{min(s,d)}}{\Phi_{min(s,d)}}\right) - F_{sd} * W_s - F_{sd} * W_s * (Z_{sd} - 1) = 0 \tag{A.2.21}$$

where $Q_{min(s,d)}$ is the sales volume on the sd-link of a variety with minimum productivity for the link. (A.2.21) reduces to MM equation (T1.4a).

MM equation (T1.4b) can be derived from (A.2.15), (A.2.16) and (T1.1b):

$$\frac{Q_{min(s,d)}}{Q_{\bullet sd}} = \left(\frac{P_{min(s,d)}}{P_{\bullet sd}}\right)^{-\sigma} = \left(\frac{\Phi_{min(s,d)}}{\Phi_{\bullet sd}}\right)^{\sigma} = \frac{1}{\beta^{\sigma}} \tag{A.2.22}$$

Deriving MM equation (T1.5)

Substituting from (A.2.19) into encompassing equation (T1.5) and then using (T1.2b), (A2.3), (A.2.15) and (A.2.16) we obtain

$$\Pi tot_s = \sum_d \sum_{k \in S(s,d)} N_{sd} \Phi_{min(sd)}^{\alpha} \alpha \Phi_k^{-\alpha-1} \left[\left(M_d - 1\right) * \frac{W_s}{\Phi_k} Q_{\bullet sd} \left(\frac{\Phi_k}{\Phi_{\bullet sd}}\right)^{\sigma} - F_{sd} W_s\right] - N_s H_s W_s \tag{A.2.23}$$

Converting to continuous form gives

$$\begin{aligned} \Pi tot_s = {} & \sum_d N_{sd} \Phi_{min(sd)}^{\alpha} \alpha \left(M_d - 1\right) W_s Q_{\bullet sd} \Phi_{\bullet sd}^{-\sigma} \int_{\Phi_{min(sd)}}^{\infty} \Phi^{-\alpha-1+\sigma-1}\, d\Phi \\ & - \sum_d N_{sd} F_{sd} W_s \Phi_{min(sd)}^{\alpha} \alpha \int_{\Phi_{min(sd)}}^{\infty} \Phi^{-\alpha-1}\, d\Phi \quad - N_s H_s W_s \end{aligned} \tag{A.2.24}$$

Performing the integrations and using (A.2.10) and (T1.1b), we can reduce (A.2.24) to MM equation (T1.5).

Deriving MM equation (T1.6)

Substituting from (T1.2b), (A.2.3), (A.2.15) and (A.2.16) into encompassing equation (T1.6) we obtain

$$L_s = \sum_d \sum_{k\in S(s,d)} N_{sd}\Phi^{\alpha}_{\min(sd)}\alpha\Phi_k^{-\alpha-1}\frac{Q_{\bullet sd}}{\Phi_k}\left(\frac{\Phi_k}{\Phi_{\bullet sd}}\right)^{\sigma} + \sum_d \sum_{k\in S(s,d)} N_{sd}\Phi^{\alpha}_{\min(sd)}\alpha\Phi_k^{-\alpha-1}F_{sd} \quad + N_s H_s \qquad \text{(A.2.25)}$$

Converting to continuous form gives

$$L_s = \sum_d N_{sd}\Phi^{\alpha}_{\min(sd)}\alpha Q_{\bullet sd}\Phi^{-\sigma}_{\bullet sd}\int_{\Phi_{\min(sd)}}^{\infty}\Phi^{-\alpha-1+\sigma-1}\,d\Phi \quad + \sum_d N_{sd}\Phi^{\alpha}_{\min(sd)}\alpha F_{sd}\int_{\Phi_{\min(sd)}}^{\infty}\Phi^{-\alpha-1}\,d\Phi \quad + N_s H_s$$

$$\text{(A.2.26)}$$

Performing the integrations and using (A.2.10) and (T1.1b), we can reduce (A.2.26) to MM equation (T1.6).

A.3. Notes on the competition weights κ(s,d) and the modifying factor for minimum productivity (Z_{sd})

Unlike the first 6 groups of MM equations in Table 1 of the main paper which are derived from encompassing equations specified at the variety level, the SGMC equations [the 7th group, (T1.7a) - (T1.7f)] are specified directly at the country and industry levels. Justifications for these equations were given in section 2 of the main paper. In this section, we provide further explanations of (T1.7c) and (T1.7e).

MM equation (T1.7c): setting values for κ(s,d)

In telling the story of the MM model we can define $Ntot_d$ as the number of firms competing in d's Widget market. However, as explained in the main text, this definition cannot be taken literally. $Ntot_d$ is an indicator variable of competition in d's Widget market. Given judgements or data on markup factors and substitution elasticities, we can derive an initial value for $Ntot_d$ from (T1.7a) and (T1.7b). For example, if we impose data or judgements suggesting that $M_d = 1.667$ and $\sigma = 5$ then $\Gamma_d = 2.5$ and $Ntot_d = 4$.

With its initial value in place, we need to specify how $Ntot_d$ moves in response to changes in the number of producing firms in all countries, N_s for all s. This is done in (T1.7c). For understanding (T1.7c) it is useful to work with the percentage change version:

$$ntot_d = \sum_s \kappa(s,d) * n_s \qquad \text{(A.3.1)}$$

where

$ntot_d$ and n_s are the percentage changes away from their initial levels in $Ntot_d$ and N_s, and the κ(s,d) parameters are weights satisfying $\sum_s \kappa(s,d) = 1$.

In setting values for the κ(s,d)s, we require a formula that recognises that if there are many firms in country s, then these can potentially provide a lot of competition in d but only if these firms are major participants in d's Widget market. We measure the extent of s's participation in d's market by $\overline{S}_{sd}$ where this is the initial share of Widget sales in d sourced from s. Combining the ideas that the contribution by s to competition in d depends on the number of firms in s and their participation in market d, we set $\kappa(s,d)$ according to

$$\kappa(s,d) = \frac{\bar{S}_{sd} * \bar{N}_s}{\sum_q \bar{S}_{qd} * \bar{N}_q} \tag{A.3.2}$$

where

$\bar{N}_s$ is the initial number of firms in country s.

Trade share data for implementing (A.3.2) are readily available from GTAP. But what about $\bar{N}_s$? In (A.3.2), only relative values are required for the N_ss. For our GTAP-MM simulations we use output values. Thus, we assume that in MM industries there is uniformity across countries in output per firm.

MM equation (T1.7e)

Melitz assumes that Widget firms in country s decide whether a variety k should be sent on the sd-link without considering the effects of their decisions on the profitability of other varieties on the link. This assumption is reasonable for Melitz' LGMC model in which there are many firms in country s and each firm produces only one variety. However, it is not an appropriate assumption for an SGMC model in which there are few firms and each firm produces many varieties. In such a model, we need to recognize that in making decisions on varieties, firms will take account of effects on sales of their own established varieties and those of other firms, and the likely reactions of other firms.

To introduce inter-variety dependencies into the specification of firm decision making, we assume that the minimum-productivity variety on the sd-link is chosen to maximize profits on the link. With the profit-maximizing variety decision in place for the link, each firm assumes that any movement it makes away from that decision will be matched by other firms, reducing its own sd-profits as well as those of its rivals. (Recall that we assume that Widget firms in s are identical.)

The first step towards deriving the profit-maximizing variety specification for the sd-link is to define total profits on the link:

$$\Pi_{tot(sd)} = \sum_{k \in S(s,d)} \Pi_{ksd} N_s B_s g_s(\Phi_k) \tag{A.3.3}$$

Substitute from (A.2.3) and (A.2.20) and adopt a continuous representation of varieties:

$$\begin{aligned}\Pi_{tot(sd)} = {} & (M_d - 1) W_s Q_d \delta_{sd}^{\sigma} P_d^{\sigma} (W_s T_{sd} M_d)^{-\sigma} N_s B_s \alpha \int_{\Phi_{min(s,d)}}^{\infty} \Phi^{\sigma-1-\alpha-1} \, d\Phi \\ & - F_{sd} W_s N_s B_s \alpha \int_{\Phi_{min(s,d)}}^{\infty} \Phi^{-\alpha-1} \, d\Phi \end{aligned} \tag{A.3.4}$$

Now we start working on the evaluation of the partial derivative of $\Pi_{tot(sd)}$ with respect to $\Phi_{min(s,d)}$. In evaluating the partial derivative, Widget firms in country s assume that $\Phi_{min(s,d)}$ does not affect wage rates, markups, number of firms, preferences and tariff and transport rates. Thus,

$$\begin{aligned}\frac{\partial \Pi_{tot(sd)}}{\partial \Phi_{min(s,d)}} = {} & -(M_d - 1) W_s Q_d \delta_{sd}^{\sigma} P_d^{\sigma} (W_s T_{sd} M_d)^{-\sigma} N_s B_s \alpha \Phi_{min(s,d)}^{\sigma-1-\alpha-1} + F_{sd} W_s N_s B_s \alpha \Phi_{min(s,d)}^{-\alpha-1} \\ & + (M_d - 1) W_s \delta_{sd}^{\sigma} (W_s T_{sd} M_d)^{-\sigma} N_s B_s \alpha \left(-\frac{\Phi_{min(s,d)}^{\sigma-1-\alpha}}{\sigma - 1 - \alpha} \right) \frac{\partial \left(P_d^{\sigma} Q_d \right)}{\partial \Phi_{min(s,d)}} \end{aligned} \tag{A.3.5}$$

If like Melitz we adopt a LGMC framework, then it is appropriate to assume that $\partial\left(P_d^{\sigma}Q_d\right)/\partial\Phi_{min(s,d)}$ equals zero. Under this assumption, optimization with respect to $\Phi_{min(s,d)}$ requires that

$$0=\left[-(M_d-1)Q_d\delta_{sd}^{\sigma}P_d^{\sigma}(W_sT_{sd}M_d)^{-\sigma}\Phi_{min(s,d)}^{\sigma-1} \quad + \quad F_{sd}\right]W_sN_sB_s\alpha\Phi_{min(s,d)}^{-\alpha-1} \tag{A.3.6}$$

Using encompassing equations (T1.1a) and (T1.3a), we can reduce (A3.6) to

$$0=\left[-(M_d-1)Q_{min(s,d)}\Phi_{min(s,d)}^{-1} \quad + \quad F_{sd}\right] \tag{A.3.7}$$

This is the Melitz equation for determining $\Phi_{min(s,d)}$.

However, in our SGMC framework, we assume that in making their variety decision for market d, firms in country s gauge how these decisions will affect prices (P_d) and quantities (Q_d). To simplify the specification of the variety decision, we lock P_d and Q_d together by assuming, as earlier, that suppliers to country d perceive an elasticity of demand in d of Γ_d. Thus, they perceive the relationship between P_d and Q_d as

$$Q_d = P_d^{-\Gamma_d} \tag{A.3.8}$$

where quantity units have been chosen so that we can omit the factor of proportionality.

Under (A.3.8), we can complete the evaluation of $\partial\Pi_{tot(sd)}/\partial\Phi_{min(s,d)}$ in (A.3.5) by obtaining a formula for $\partial\left(P_d^{\sigma-\Gamma_d}\right)/\partial\Phi_{min(s,d)}$.

We start the task of evaluating $\partial\left(P_d^{\sigma-\Gamma_d}\right)/\partial\Phi_{min(s,d)}$ by working on encompassing equation (T1.2a). Substituting from (A.2.3) and (T1.1a) into (T1.2a) gives

$$P_d^{1-\sigma}=\sum_r \int_{\Phi_{min(r,d)}}^{\infty} N_rB_r\alpha\delta_{rd}^{\sigma}(W_rT_{rd}M_d)^{1-\sigma}\Phi^{\sigma-1-\alpha-1}\,d\Phi \tag{A.3.9}$$

Performing the integration and using (A.2.10) we find that

$$P_d^{1-\sigma}=\sum_r N_rB_r\beta^{\sigma-1}\delta_{rd}^{\sigma}(W_rT_{rd}M_d)^{1-\sigma}\Phi_{min(r,d)}{}^{\sigma-1-\alpha} \tag{A.3.10}$$

Differentiating with respect to $\Phi_{min(s,d)}$ in (A.3.10) leads to

$$\frac{\partial P_d}{\partial\Phi_{min(s,d)}} = P_d{}^{\sigma}N_sB_s\beta^{\sigma-1}\delta_{sd}^{\sigma}(W_sT_{sd}M_d)^{1-\sigma}\left(\frac{\alpha-(\sigma-1)}{\sigma-1}\right)\Phi_{min(s,d)}{}^{\sigma-1-\alpha-1} \tag{A.3.11}$$

In deriving (A.3.11) we hold constant $\Phi_{min(r,d)}$ for $r \neq s$. This is consistent with all firms supplying to the d market having reached a Nash equilibrium with respect to variety decisions.

Recalling from (A.2.8) that $\alpha > \sigma$-1, and that $\sigma > 1$, we see from (A3.11) that $\partial P_d/\partial\Phi_{min(s,d)}$ is positive. An increase in $\Phi_{min(s,d)}$ reduces the varieties available to Widget users in d. Through the love-of-variety effect, this increases the cost in d of satisfying a unit of Widget demand.

Next we note that

$$\frac{\partial P_d{}^{\sigma-\Gamma_d}}{\partial\Phi_{min(s,d)}} = \left(\sigma-\Gamma_d\right)P_d^{\sigma-\Gamma_d-1}\frac{\partial P_d}{\partial\Phi_{min(s,d)}} \tag{A.3.12}$$

Bringing in (A.3.11) and using (A.2.10) gives

$$\frac{\partial P_d{}^{\sigma-\Gamma_d}}{\partial\Phi_{min(s,d)}} = \left(\sigma-\Gamma_d\right)P_d^{2\sigma-\Gamma_d-1} * N_sB_s\delta_{sd}^{\sigma}(W_sT_{sd}M_d)^{1-\sigma}\left(\frac{\alpha}{\sigma-1}\right)\Phi_{min(s,d)}{}^{\sigma-1-\alpha-1} \tag{A.3.13}$$

Now substitute (A.3.13) into (A.3.5) and assume, consistent with optimization, that $\partial\Pi_{tot(sd)} / \partial\Phi_{min(s,d)}$ is zero:

$$0 = -(M_d - 1) W_s Q_d \delta_{sd}^{\sigma} P_d^{\sigma} (W_s T_{sd} M_d)^{-\sigma} N_s B_s \alpha \Phi_{min(s,d)}^{\sigma-1-\alpha-1} \quad + \quad F_{sd} W_s N_s B_s \alpha \Phi_{min(s,d)}^{-\alpha-1}$$

$$+(M_d - 1) W_s \delta_{sd}^{\sigma} (W_s T_{sd} M_d)^{-\sigma} N_s B_s \alpha \left(-\frac{\Phi_{min(s,d)}^{\sigma-1-\alpha}}{\sigma - 1 - \alpha} \right) \tag{A.3.14}$$

$$* \left[(\sigma - \Gamma_d) P_d^{2\sigma-\Gamma_d-1} * N_s B_s \delta_{sd}^{\sigma} (W_s T_{sd} M_d)^{1-\sigma} \left(\frac{\alpha}{\sigma - 1} \right) \Phi_{min(s,d)}^{\sigma-1-\alpha-1} \right]$$

Dividing through by $W_s N_s B_s \alpha \Phi_{min(s,d)}^{-\alpha-1}$ and using (A.3.8), we arrive at

$$0 = -(M_d - 1) Q_d \delta_{sd}^{\sigma} P_d^{\sigma} (W_s T_{sd} M_d)^{-\sigma} \Phi_{min(s,d)}^{\sigma-1} \quad + \quad F_{sd}$$

$$+ (M_d - 1) Q_d \delta_{sd}^{\sigma} (W_s T_{sd} M_d)^{-\sigma} \left(-\frac{\Phi_{min(s,d)}^{\sigma}}{\sigma - 1 - \alpha} \right) \alpha P_d^{2\sigma-1} N_s B_s \delta_{sd}^{\sigma} (W_s T_{sd} M_d)^{1-\sigma} \left(\frac{\sigma - \Gamma_d}{\sigma - 1} \right) \Phi_{min(s,d)}^{\sigma-1-\alpha-1} \tag{A.3.15}$$

Using (T1.1b) and (A.2.10) we obtain

$$0 = -(M_d - 1) Q_d \delta_{sd}^{\sigma} P_d^{\sigma} \left(\frac{W_s T_{sd} M_d}{\Phi_{\bullet sd}} \right)^{-\sigma} \beta^{-\sigma} \Phi_{min(s,d)}^{-1} \quad + \quad F_{sd}$$

$$+ (M_d - 1) Q_d \delta_{sd}^{\sigma} \left(\frac{W_s T_{sd} M_d}{\Phi_{\bullet sd}} \right)^{-\sigma} P_d^{2\sigma-1} N_s B_s \delta_{sd}^{\sigma} \left(\frac{W_s T_{sd} M_d}{\Phi_{\bullet sd}} \right)^{1-\sigma} \left(\frac{\sigma - \Gamma_d}{\sigma - 1} \right) \beta^{-\sigma} \Phi_{min(s,d)}^{-\alpha-1} \tag{A.3.16}$$

Substitute from (T1.3a) and (T1.1a):

$$0 = -(M_d - 1) Q_{\bullet sd} \beta^{-\sigma} \Phi_{min(s,d)}^{-1} \quad + \quad F_{sd}$$

$$+ (M_d - 1) Q_{\bullet sd} P_d^{\sigma-1} N_s B_s \delta_{sd}^{\sigma} P_{\bullet sd}^{1-\sigma} \left(\frac{\sigma - \Gamma_d}{\sigma - 1} \right) \beta^{-\sigma} \Phi_{min(s,d)}^{-\alpha-1} \tag{A.3.17}$$

Use (T1.2b):

$$0 = -(M_d - 1) Q_{\bullet sd} \beta^{-\sigma} \Phi_{min(s,d)}^{-1} \quad + \quad F_{sd}$$

$$+ (M_d - 1) Q_{\bullet sd} P_d^{\sigma-1} N_{sd} \delta_{sd}^{\sigma} P_{\bullet sd}^{1-\sigma} \left(\frac{\sigma - \Gamma_d}{\sigma - 1} \right) \beta^{-\sigma} \Phi_{min(s,d)}^{-1} \tag{A.3.18}$$

Now we start introducing values. Substituting from (T1.3b) into (A.3.18) gives

$$0 = -(M_d - 1) Q_{\bullet sd} \beta^{-\sigma} \Phi_{min(s,d)}^{-1} \quad + \quad F_{sd} + \quad (M_d - 1) P_d^{\sigma-1} V_{sd} \delta_{sd}^{\sigma} P_{\bullet sd}^{-\sigma} \left(\frac{\sigma - \Gamma_d}{\sigma - 1} \right) \beta^{-\sigma} \Phi_{min(s,d)}^{-1} \tag{A.3.19}$$

Then via (T1.4b) and (T1.3a) we have

$$0 = -(M_d - 1) Q_{min(s,d)} \Phi_{min(s,d)}^{-1} \quad + \quad F_{sd} + (M_d - 1) \frac{V_{sd}}{P_d Q_d} \left(\frac{\sigma - \Gamma_d}{\sigma - 1} \right) Q_{min(s,d)} \Phi_{min(s,d)}^{-1} \tag{A.3.20}$$

which can be written as

$$(M_d - 1)\frac{Q_{\min(s,d)}}{\Phi_{\min(s,d)}} = \frac{F_{sd}}{\left(1 - \frac{V_{sd}}{P_d Q_d}\left(\frac{\sigma - \Gamma_d}{\sigma - 1}\right)\right)} \qquad \text{(A.3.21)}$$

This establishes (T1.7e) as the form of the Z factor appearing in (T1.4a).

A.4. Preparing the MM model for implementation in GEMPACK

The GEMPACK software[1] can accept equations presented as relationships between levels of variables. However, the software works most conveniently when the equations are presented as linear relationships between percentage-change or change variables where these are deviations from an initial equilibrium.

In this section, we start by converting the MM equations from Table 1 of the main paper into linear deviation form. Then we show how MM assumptions can be embedded in a standard GEMPACK version of GTAP by closure swaps and the addition of a few equations, with minimal changes to the initial model.

A linear deviation representation of the MM equations

The left column of Table A.2 repeats the MM equations from Table 1 of the main paper. Linear deviation versions of these equations, suitable for GEMPACK, are in the centre column. In the left column, variables are depicted as uppercase symbols. In the centre column, we use corresponding lowercase symbols for percentage deviations and "d" for ordinary changes. For example, $p_{\bullet sd}$ is the percentage deviation in $P_{\bullet sd}$, and $d\Pi tot_s$ is the ordinary change in Πtot_s.[2]

For most of the MM equations, the conversion from the left column of Table A.2 to the centre column involves straightforward applications of total differentiation.[3] Here we focus on the conversions for (T1.2a), (T1.5) and (T1.6). In the derivation of the linear deviation forms for these equations, we undertake extra steps designed to facilitate the evaluation of coefficients via easily accessed data items such as sales shares.

Derivation of the linear deviation form for (T1.2a)

From MM equation (T1.2a) in the left panel of Table A.2, we obtain

$$(1-\sigma) * p_d = \sum_s \left(\frac{N_{sd}\delta_{sd}^{\sigma} P_{\bullet sd}^{1-\sigma}}{\sum_r N_{rd}\delta_{rd}^{\sigma} P_{\bullet rd}^{1-\sigma}} \right) * \left[n_{sd} + (1-\sigma) * p_{\bullet sd} \right] \qquad \text{(A.4.1)}$$

Using MM equation (T1.3a), we can write the complicated share coefficient on the RHS of (A.4.1) as

$$\left(\frac{N_{sd}\delta_{sd}^{\sigma} P_{\bullet sd}^{1-\sigma}}{\sum_r N_{rd}\delta_{rd}^{\sigma} P_{\bullet rd}^{1-\sigma}} \right) = \frac{N_{sd} P_{\bullet sd} Q_{\bullet sd} \left(Q_d^{-1} \delta_{sd}^{-\sigma} P_d^{-\sigma} P_{\bullet sd}^{\sigma} \right) \delta_{sd}^{\sigma} P_{\bullet sd}^{-\sigma}}{\sum_r N_{rd} P_{\bullet rd} Q_{\bullet rd} \left(Q_d^{-1} \delta_{rd}^{-\sigma} P_d^{-\sigma} P_{\bullet rd}^{\sigma} \right) \delta_{rd}^{\sigma} P_{\bullet rd}^{-\sigma}} \qquad \text{(A.4.2)}$$

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[1] See Horridge et al. (2013, 2018).

[2] We prefer to work in percentage deviations. But for some variables such as profits, this is not possible because their values can pass through zero.

[3] For a brief introduction to the derivation of deviation equations for use in GEMPACK, see Dixon et al. (2018, box 6.1). This reference also describes how GEMPACK updates the coefficients in the linear deviation equations so that accurate solutions to the initial non-linear model are produced in a multi-step process.

Via (T1.3b), the RHS of (A.4.2) reduces to $V_{sd} \big/ \sum_r V_{rd}$. From here, we quickly arrive at the linear deviation form for (T1.2a) shown in the centre column of Table A.2.

Table A.2. MM equations in levels and linear deviation forms.

	Levels	Percentage change version suitable for GEMPACK	Role in GEMPACK implementation of MM
(T1.1a)	$P_{\bullet sd} = \left(\frac{W_s T_{sd}}{\Phi_{\bullet sd}}\right) * M_d$	$p_{\bullet sd} = \left(w_s + t_{sd} - \phi_{\bullet sd}\right) + m_d$	Omitted
(T1.1b)	$\Phi_{\bullet sd} = \beta * \Phi_{\min(s,d)}$	$\phi_{\bullet sd} = \phi_{\min(s,d)}$	Determines $\phi_{\bullet sd}$
(T1.2a)	$P_d = \left(\sum_s N_{sd} \delta_{sd}^{\sigma} P_{\bullet sd}^{1-\sigma}\right)^{\frac{1}{(1-\sigma)}}$	$p_d = \sum_s S_{sd} * \left[p_{\bullet sd} - \frac{1}{\sigma - 1} * n_{sd}\right]$	Omitted
(T1.2b)	$N_{sd} = N_s * B_s * \left(\Phi_{\min(s,d)}\right)^{-\alpha}$	$n_{sd} = n_s - \alpha * \phi_{\min(s,d)}$	Determines n_{sd}
(T1.3a)	$Q_{\bullet sd} = Q_d \delta_{sd}^{\sigma} \left(\frac{P_d}{P_{\bullet sd}}\right)^{\sigma}$	$q_{sd} - n_{sd} = q_d - \sigma * (p_{\bullet sd} - p_d)$	Omitted
(T1.3b)	$V_{sd} = P_{\bullet sd} N_{sd} Q_{\bullet sd}$	$v_{sd} = p_{\bullet sd} + n_{sd} + q_{\bullet sd}$	Omitted
(T1.3c)	$Q_{sd} = N_{sd} Q_{\bullet sd}$	$q_{sd} = n_{sd} + q_{\bullet sd}$	Determines $q_{\bullet sd}$
(T1.4a)	$(M_d - 1)\left(\frac{Q_{\min(s,d)}}{\Phi_{\min(s,d)}}\right) - F_{sd} * Z_{sd} = 0$	$\left(\frac{M_d}{M_d - 1}\right) * m_d + q_{\min(s,d))} - \phi_{\min(s,d))} = z_{sd} + f_{sd}$	Determines $\phi_{\min(s,d))}$
(T1.4b)	$Q_{\min(s,d)} = Q_{\bullet sd} / \beta^{\sigma}$	$q_{\min(s,d)} = q_{\bullet sd}$	Determines $q_{\min(s,d)}$

(Continued…)

Table A.2. MM equations in levels and linear deviation forms. (…Continued)

	Levels	Percentage change version suitable for GEMPACK	Role in GEMPACK implementation of MM
(T1.5)	$\Pi tot_s = \sum_d N_{sd}\left[\frac{W_s}{\Phi_{\bullet sd}} * Q_{\bullet sd} * (M_d - 1) - W_s * F_{sd}\right] - N_s H_s W_s$	$100 * d\Pi tot_s = \sum_d \left[\left(1 - \frac{1}{M_d}\right)\right] * MARKETV(s,d) * (n_{sd} + w_s - \phi_{\bullet sd} + q_{\bullet sd})$ $+ \sum_d MARKETV(s,d) * m_d$ $- \sum_d \left(\frac{(M_d - 1)}{M_d \beta^{\sigma-1} Z_{sd}}\right) * MARKETV(s,d) * (w_s + n_{sd} + f_{sd})$ $- \sum_d \left[\left(1 - \frac{1}{M_d}\right) - \left(\frac{(M_d - 1)}{M_d \beta^{\sigma-1} Z_{sd}}\right)\right] * MARKETV(s,d) * (w_s + n_s + h_s)$ $+ \Pi tot_s * (w_s + n_s + h_s)$	Determines $d\Pi tot_s$
(T1.6)	$L_s = \sum_d \frac{N_{sd} Q_{\bullet sd}}{\Phi_{\bullet sd}} + \sum_d N_{sd} F_{sd} + N_s H_s$	$W_s L_s * (w_s + \ell_s) = \sum_d \frac{1}{M_d} * MARKETV(s,d) * (w_s + n_{sd} + q_{\bullet sd} - \phi_{\bullet sd})$ $+ \sum_d \left(\frac{(M_d - 1)}{M_d \beta^{\sigma-1} Z_{sd}}\right) * MARKETV(s,d) * (w_s + n_{sd} + f_{sd})$ $+ \sum_d \left[\left(1 - \frac{1}{M_d}\right) - \left(\frac{(M_d - 1)}{M_d \beta^{\sigma-1} Z_{sd}}\right)\right] * MARKETV(s,d) * (w_s + n_s + h_s)$ $- \Pi tot_s * (w_s + n_s + h_s)$	Determines ao_s

(Continued…)

Table A.2. MM equations in levels and linear deviation forms. (…Continued)

	Levels	Percentage change version suitable for GEMPACK	Role in GEMPACK implementation of MM
Add-ons for small group monopolistic competition			
(T1.7a)	$M_d = \dfrac{\Gamma_d}{\Gamma_d - 1}$	$m_d = \dfrac{-1}{\Gamma_d - 1} * \lambda_d$	Determines m_d
(T1.7b)	$\Gamma_d = \dfrac{1}{\dfrac{1}{Ntot_d} + \left(1 - \dfrac{1}{Ntot_d}\right) * \dfrac{1}{\sigma}}$	$\lambda_d = \dfrac{\sigma - 1}{\sigma - 1 + Ntot_d} * ntot_d$	Determines λ_d
(T1.7c)	$Ntot_d = \left(\dfrac{\bar{N}tot_d}{\prod_s \bar{N}_s^{K(s,d)}}\right) * \prod_s N_s^{K(s,d)}$	$ntot_d = \sum_s \kappa(s,d) * n_s$	Determines $ntot_d$
(T1.7d)	$S_{sd} = \dfrac{V_{sd}}{P_d Q_d}$	$s_{sd} = v_{sd} - (p_d + q_d)$	Determines s_{sd}
(T1.7e)	$Z_{sd} = 1 \Big/ \left(1 - S_{sd} * \left(\dfrac{\sigma - \Gamma_d}{\sigma - 1}\right)\right)$	$z_{sd} = \dfrac{Z_{sd} S_{sd}}{\sigma - 1} * \left[(\sigma - \Gamma_d) * s_{sd} - \Gamma_d * \lambda_d\right]$	Determines z_{sd}
(T1.7f)	$N_s = \bar{N}_s * \exp\left(\Psi_{1s}\left(\dfrac{\Pi tot_s}{W_s L_s} - \Psi_{0s}\right)\right)$	$n_s = \dfrac{\Psi_{1s}}{W_s L_s} * \left[100 * d\Pi tot_s - \Pi tot_s * (w_s + \ell_s)\right] + 100 * \left(\dfrac{\Pi tot_s}{W_s L_s} - \Psi_{0s}\right) * d\Psi_{1s} - 100 * \Psi_{1s} * d\Psi_{0s}$	Determines n_s

Derivation of the linear deviation form for (T1.6)

We start by multiplying the MM equation (T1.6) through by W_s:

$$W_s L_s = \sum_d \frac{N_{sd} W_s Q_{\bullet sd}}{\Phi_{\bullet sd}} + \sum_d N_{sd} W_s F_{sd} \quad + N_s W_s H_s \tag{A.4.3}$$

(A.4.3) can be presented in linear deviation form as:

$$\begin{aligned} W_s L_s * (w_s + \ell_s) = \sum_d \frac{N_{sd} W_s Q_{\bullet sd}}{\Phi_{\bullet sd}} * (n_{sd} + w_s + q_{\bullet sd} - \phi_{\bullet sd}) \\ + \sum_d N_{sd} W_s F_{sd} * (n_{sd} + w_s + f_{sd}) \quad + N_s W_s H_s * (n_s + w_s + h_s) \end{aligned} \tag{A.4.4}$$

We could stop at (A.4.4). However, for setting initial values of coefficients, it is helpful to perform some additional steps.

Using (T1.1a), we see that

$$\frac{N_{sd} W_s Q_{\bullet sd}}{\Phi_{\bullet sd}} = \frac{N_{sd}\ Q_{\bullet sd} P_{\bullet sd}}{T_{\bullet sd}} * \frac{1}{M_d} \tag{A.4.5}$$

Applying (T1.3b) gives

$$\frac{N_{sd} W_s Q_{\bullet sd}}{\Phi_{\bullet sd}} = \mathrm{MARKETV}(s,d) * \frac{1}{M_d} \tag{A.4.6}$$

where

$$\mathrm{MARKETV}(s,d) = \frac{V_{sd}}{T_{\bullet sd}} \tag{A.4.7}$$

MARKETV(s,d) is the factory-door value of the Widget flow from s to d. It excludes transport costs and tariffs which are reflected in $T_{\bullet sd}$.

Using (T1.1a) and (T1.4a) we see that

$$N_{sd} W_s F_{sd} = \frac{N_{sd} Q_{\bullet sd} P_{\bullet sd}}{T_{sd}} * \frac{1}{M_d} * \frac{\Phi_{\bullet sd}}{Q_{\bullet sd}} * \frac{(M_d - 1)}{Z_{sd}} * \frac{Q_{\min(s,d)}}{\Phi_{\min(s,d)}} \tag{A.4.8}$$

Via (T1.3b), (A.4.7), (T1.1b) and (T1.4b), we find that

$$N_{sd} W_s F_{sd} = \mathrm{MARKETV}(s,d) * \frac{(M_d - 1)}{M_d \beta^{\sigma-1} Z_{sd}} \tag{A.4.9}$$

Using (T1.5) together with (A.4.5) and (A.4.9), we see that

$$\begin{aligned} N_s W_s H_s = \sum_d \mathrm{MARKETV}(s,d) * \frac{(M_d - 1)}{M_d} \\ - \sum_d \mathrm{MARKETV}(s,d) * \frac{(M_d - 1)}{M_d \beta^{\sigma-1} Z_{sd}} \quad - \Pi tot_s \end{aligned} \tag{A.4.10}$$

Substituting from (A.4.6), (A.4.9) and (A.4.10) into (A.4.4) leads to the linear deviation form for (T1.6) given in the centre column of Table A.2.

Notice that (A.4.6), (A.4.9) and (A.4.10) reveal the split of revenue [MARKETV(s,d)] from the sd-link:

- the fraction $1/M_d$ covers the cost of producing the Widgets sent on the link;
- the fraction $\dfrac{(M_d - 1)}{M_d \beta^{\sigma-1} Z_{sd}}$ covers the fixed costs of setting up the link; and
- the remaining fraction, $1 - \dfrac{1}{M_d} - \dfrac{(M_d - 1)}{M_d \beta^{\sigma-1} Z_{sd}}$, contributes to covering the fixed costs of setting up Widget firms in s ($N_s H_s W_s$) and to pure profits (Πtot_s).

Understanding this split is helpful in calibrating the model. In choosing starting values (values in the initial equilibrium) for M_d and Z_{sd} and in fixing the values for the parameters underlying β, we can make sure that the implied revenue splits accord with prior judgements.

Derivation of the linear deviation form for (T1.5)

From MM equation (T1.5), we have

$$\begin{aligned} 100 * d\Pi tot_s = & \sum_d \frac{N_{sd} W_s Q_{\bullet sd}}{\Phi_{\bullet sd}} * (M_d - 1)(n_{sd} + w_s + q_{\bullet sd} - \phi_{\bullet sd}) \\ & + \sum_d \frac{N_{sd} W_s Q_{\bullet sd}}{\Phi_{\bullet sd}} * (M_d - 1) * \frac{M_d}{(M_d - 1)} * m_{sd} \\ & - \sum_d N_{sd} W_s F_{sd} * (n_{sd} + w_s + f_{sd}) \quad - N_s W_s H_s * (n_s + w_s + h_s) \end{aligned} \tag{A.4.11}$$

Using (A.4.6), (A.4.9) and (A.4.10), we can re-write (A.4.11) in the form shown in the centre column of Table A.2 for the linear deviation version of (T1.5)

A.5. Making minimal alterations to standard GTAP to accommodate an MM industry

In earlier work[4], we showed how Melitz assumptions can be introduced to GTAP. Our method involves the addition of a few equations with minimal alterations to the core model. Here we describe a similar method for MM.

To simplify the exposition, we continue to assume (as in Table A.2) that we are dealing with a model in which (a) labor is the only primary factor and (b) all agents (firms, households and government) in country d use Widgets from different sources in the same proportions, thus requiring only one Widget-mixer for each country. Although these two simplifications do not apply to standard GTAP, we explain our MM method as though they do. The GEMPACK code that we can supply with this paper copes with both these complications.

Starting from standard GTAP, we can create an MM industry by adding the equations from the centre column of Table A.2 to the end of the model, but excluding:

(T1.1a) and (T1.2a) which are MM specifications of the price to Widget users in d of the typical variety from s and the price to consumers in d of a composite unit of Widgets;

and

(T1.3a) and (T1.3b), which are MM specifications of demands and values in d for Widgets from s.

As shown in the right column of Table A.2, we can think of these additional equations as determining all the new variables that are required for an MM industry but which are not part of standard GTAP. The only non-intuitive entry in the third column of Table A.2 is ao_s entered against equation (T1.6). This is the percentage change in a shift variable that moves the ratio

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[4] See Dixon *et al.* (2018 and 2019). A similar approach is described in Bekkers and Francois (2018).

of output to input in s's Widget industry. As in standard GTAP, it is an all-input-saving technical change. If ao_s = 10, then the Widget industry in s can produce any given level of output with 10 per cent less of all inputs. For an MM industry, ao_s moves endogenously to reconcile the determination of total inputs to the Widget industry given by (T1.6) with the GTAP specification in which total input (L_s) is output deflated by total factor productivity (QO_s/AO_s).

But what about the omitted equations: why aren't they added to the end of the GTAP equations along with the other MM equations?

The answer to this question is that standard GTAP already has equations that determine $p_{\bullet sd}$, p_d, q_{sd} and v_{sd}. If we simply included (T1.1a), (T1.2a), (T1.3a) and (T1.3b) as add-ons to GTAP, they would clash with existing GTAP equations.

With these equations not appearing explicitly in our model, how can we ensure that they are satisfied?

We do this by adding equations that drive naturally exogenous tax and preference variables. The equations cause the GTAP equations for $p_{\bullet sd}$, p_d, q_{sd} and v_{sd} to generate results compatible with (T1.1a), (T1.2a), (T1.3a) and (T1.3b). The equations we add are:

$$tx_{sd} = ao_s - \phi_{\bullet sd} + m_d \qquad (A.5.1)$$

and

$$aa_{sd} = \frac{1}{(\sigma - 1)} * n_{sd} \qquad (A.5.2)$$

In these equations:

tx_{sd} is the percentage change in the power of a tax on the flow of Widgets from s to d (includes s to s). This is charged in country s at the factory door and is included in the market value of the sd flow. tx_{sd} is not part of standard GTAP but it is easily added as a destination-specific shifter in the GTAP specification of export taxes.

ao_s is, as mentioned above, the percentage change in a shift variable that moves the ratio of output to input in s's Widget industry.

aa_{sd} is the percentage change in a shift variable that moves the preferences/technology of the Widget mixer in country s. As in standard GTAP, it is an s-saving change. If aa_{sd} = 10, then the mixer who provides composite Widgets to users in d can satisfy any given level of Widget requirements with 10 per cent less Widgets from s while holding constant Widget purchases (quantities) from other sources.

With (A.5.1) and (A.5.2), together with the included equations from the centre column of Table A.2, appended to the equations of standard GTAP, the excluded equations, [(T1.1a), (T1.2a), (T1.3a) and (T1.3b) are satisfied. This means that standard GTAP extended by the included equations and (A.5.1) – (A.5.2) produces results consistent with MM theory.

Demonstrating that (T1.1a) is satisfied

In standard GTAP, the percentage change in the price to Widget users in d of Widgets sent from s is given by

$$p_{\bullet sd} = w_s - ao_s + tx_{sd} + t_{sd} \qquad (A.5.3)$$

In (A.5.3), $p_{\bullet sd}$ is determined by the percentage change in the cost of inputs per unit of output in s's Widget industry ($w_s - ao_s$) inflated by the percentage change in the power of the sd tax imposed by country s (tx_{sd}) and further inflated by the percentage change in the power of tariffs and transport costs applying to the sd flow (t_{sd}). With tx_{sd} specified by (A.5.1), we obtain the percentage change version of (T1.1a).

Demonstrating that (T1.2a), (T1.3a) and (T1.3b) are satisfied

In standard GTAP, demands in d for Widgets from s are determined by percentage change equations consistent with:

$$q_{sd} + aa_{sd} = q_d - \sigma * \left[(p_{\bullet sd} - aa_{sd}) - p_d \right] \tag{A.5.4}$$

$$p_d = \sum_t S_{td} * (p_{\bullet td} - aa_{td}) \tag{A.5.5}$$

and

$$v_{sd} = p_{\bullet sd} + q_{sd} \tag{A.5.6}$$

Substituting from (A.5.2) into (A.5.5) gives (T1.2a). Substituting from the percentage change version of (T1.3c) into (A.5.6) gives (T1.3b). Substituting from (A.5.2) into (A.5.4) gives

$$q_{sd} + \frac{1}{(\sigma - 1)} * n_{sd} = q_d - \sigma * \left[(p_{\bullet sd} - \frac{1}{(\sigma - 1)} * n_{sd}) - p_d \right] \tag{A.5.7}$$

This simplifies to the percentage change version of (T1.3a).

Who gets the revenue from the destination-specific export taxes?

The base for the artificial tax on the sd-flow that we introduce to achieve MM pricing is MARKETV(s,d)/TX_{sd} where TX_{sd} is the level of the power of the sd tax. MARKETV(s,d) is, as before, the market value of the sd flow, that is payments by Widget users in d for Widgets delivered from s, excluding tariffs and transport costs.

Total tax revenue on the Widget flows from s is given by

$$\mathrm{CollRev}(s) = \sum_d \mathrm{MARKETV}(s,d) - \sum_d \frac{\mathrm{MARKETV}(s,d)}{\mathrm{TX}(s,d)} \tag{A.5.8}$$

The first term on the RHS of (A.5.8) is the total value of payments to Widget producers in s at the factory door. With the equations of standard GTAP continuing to apply, the second term is the cost of inputs (including production taxes) to the Widget industry in s, that is factory-door revenue excluding factory-door destination-specific taxes. The difference between these two terms is profits (which could be positive or negative). Consequently, it turns out that the artificial destination-specific taxes are returned to Widget producers in s.

We can check the computation of profits by comparing results generated by (T1.5) with those generated by (A.5.8).

A.6. Explaining how a reduction in competition can generate an increase in varieties and an increase in the costs to consumers of MM products

In the simulation described in section 3 of the main paper, we found that a reduction in world-wide competition in the 13 MM industries resulted in more varieties of MM commodities being delivered to each market. At the same time, the simulation showed increases in unit costs to MM consumers despite the cost-reducing effects of increased variety.

In this section we show that a sufficient condition for an increase in varieties and unit costs in each market d is that $Z_{sd} / (M_d\text{-}1)$ falls for all s.

In section 3 of the main paper, we explained that a reduction in competition increases both Z_{sd} and (M_d-1). A priori we could not sign the direction of movement for the ratio of Z_{sd} to

(M_d-1). However, the results showed a decline. For example, for North America the average Z_{sd} increased by 3.64 per cent (row 6, Table 2 of the main paper) while the average M_d increased by 2.56 per cent (row 5). With the initial values for the markup factors being about 2, the Z_{sd} to (M_d -1) ratios decreased by about 1.48 per cent (= 3.64 - 2*2.56).

Increase in the number of varieties delivered to d

We calculate the percentage change in the number of varieties of an MM commodity delivered to d as a share weighted average of the percentage changes in the number of varieties delivered from each source s:

$$\text{varieties(d)} = \sum_s S_{sd} * n_{sd} \tag{A.6.1}$$

where

varieties(d) is the percentage change in the number of varieties delivered to d;

S_{sd} is the share of d's expenditure on the MM commodity devoted to source s; and

n_{sd} is the percentage change in the number of varieties sent from s to d.

Given that $Z_{sd}/(M_d -1)$ falls for all s, we show that the number of varieties delivered to market d increases.

We start with equation (T1.4a) in Table 1 of the main paper. We assume that F_{sd} is constant. Then with reductions in $Z_{sd}/(M_d -1)$ for all s, we see that $Q_{\min(s,d)}/\Phi_{\min(s,d)}$ falls for all s.

From (T1.4b), (T1.3a), (T1.1a) and (T1.1b), we obtain

$$\frac{Q_{\min(s,d)}}{\Phi_{\min(s,d)}} = \left[\frac{Q_d \delta_{sd}^{\sigma} P_d^{\sigma}}{(W_s T_{sd} M_d)^{\sigma}}\right] * \Phi_{\min(s,d)}^{\sigma-1} \tag{A.6.2}$$

Working with (T1.2a) and (T1.1a) we find that

$$\frac{P_d}{M_d} = \left[\sum_k N_{kd} \delta_{kd}^{\sigma} \left(\frac{W_k T_{kd}}{\Phi_{\bullet kd}}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}} \tag{A.6.3}$$

Using (T1.1b) and (T1.2b), we obtain

$$\frac{P_d}{M_d} = \left[\sum_k N_k B_k \Phi_{\min(k,d)}^{-(\alpha-(\sigma-1))} \delta_{kd}^{\sigma} \left(\frac{W_k T_{kd}}{\beta}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}} \tag{A.6.4}$$

Substituting from (A.6.4) into (A.6.2) gives

$$\frac{Q_{\min(s,d)}}{\Phi_{\min(s,d)}} = \frac{Q_d \delta_{sd}^{\sigma}}{(W_s T_{sd})^{\sigma}} * \text{Term}(d) * \Phi_{\min(s,d)}^{\sigma-1} \tag{A.6.5}$$

where

$$\text{Term}(d) = \left[\sum_k N_k B_k \Phi_{\min(k,d)}^{-(\alpha-(\sigma-1))} \delta_{kd}^{\sigma} \left(\frac{W_k T_{kd}}{\beta}\right)^{1-\sigma}\right]^{\frac{\sigma}{1-\sigma}} \tag{A.6.6}$$

Next, we convert (A.6.5) and (A.6.6) into percentage change form. In making the conversion, we ignore changes in W_s and Q_d. These are variables whose values are determined mainly outside the industries receiving shocks (in this case reductions in competition). We expect these variables to be only mildly affected by the shocks. We assume zero changes in T_{sd}, δ_{sd}, σ, B_k and β. They are either exogenous variables or parameters. Under these assumptions, percentage change forms for (A.6.5) and (A.6.6) can be written as:

$$q_{\min(s,d)} - \phi_{\min(s,d)} = \text{term}(d) + (\sigma - 1) * \phi_{\min(s,d)} \tag{A.6.7}$$

$$\text{term}(d) = \frac{\sigma}{1-\sigma} \sum_k H_{kd} * \left\{ n_k - (\alpha - (\sigma - 1)) * \phi_{\min(k,d)} \right\} \tag{A.6.8}$$

where

$q_{\min(s,d)}$, $\phi_{\min(s,d)}$, term(d) and n_k are percentage changes in the variables denoted by the corresponding uppercase variables and

$$H_{kd} = \frac{N_k B_k \Phi_{\min(k,d)}^{-(\alpha-(\sigma-1))} \delta_{kd}^{\sigma} \left(\dfrac{W_k T_{kd}}{\beta} \right)^{1-\sigma}}{\sum_r N_r B_r \Phi_{\min(r,d)}^{-(\alpha-(\sigma-1))} \delta_{rd}^{\sigma} \left(\dfrac{W_r T_{rd}}{\beta} \right)^{1-\sigma}} \tag{A.6.9}$$

Multiplying (A.6.7) through by H_{sd} and summing over s we obtain:

$$\sum_s H_{sd} * \left(q_{\min(s,d)} - \phi_{\min(s,d)} \right) = \text{term}(d) + (\sigma - 1) * \sum_s H_{sd} * \phi_{\min(s,d)} \tag{A.6.10}$$

Now substitute (6.8) into (6.10):

$$\sum_s H_{sd} * \left(q_{\min(s,d)} - \phi_{\min(s,d)} \right) = \frac{\sigma}{1-\sigma} \sum_k H_{kd} * n_k + \left(\frac{\sigma\alpha - (\sigma - 1)}{\sigma - 1} \right) \sum_k H_{kd} \phi_{\min(k,d)} \tag{A.6.11}$$

In a simulation of the effects of a worldwide reduction in competition, we can assume that the number of firms decreases in each country. Hence $\sum_k H_{kd} * n_k$ is negative. Then because σ>1, the first term on the RHS of (A.6.11) is positive. We have already established that $q_{\min(s,d)} - \phi_{\min(s,d)}$ is negative for all s. Thus, the LHS of (A.6.11) is negative. We can conclude that the second term on the RHS is negative. Recall from (A.2.8) that $\alpha > \sigma - 1$. Hence $(\sigma\alpha - (\sigma - 1))/(\sigma - 1)$ >0 and

$$\sum_k H_{kd} \phi_{\min(k,d)} < 0 \tag{A.6.12}$$

Now we work with a percentage change version of (T1.2b)

$$n_{sd} = n_s - \alpha * \phi_{\min(s,d)} \tag{A.6.13}$$

where

n_{sd} is the percentage change in the number of varieties sent from s to d.

Multiplying (A.6.13) through by H_{sd} and summing over s we obtain:

$$\sum_s H_{sd} * n_{sd} = \sum_s H_{sd} * n_s - \alpha * \sum_s H_{sd} * \phi_{\min(s,d)} \tag{A.6.14}$$

Using (A.6.11) to eliminate $\sum_k H_{kd} * n_k$ gives

$$\sum_s H_{sd} * n_{sd} = \frac{1-\sigma}{\sigma} \sum_s H_{sd} * \left(q_{\min(s,d)} - \phi_{\min(s,d)} \right) + \left(\frac{1-\sigma}{\sigma} \right) \sum_k H_{kd} \phi_{\min(k,d)} \tag{A.6.15}$$

With (1-σ) being negative and with $q_{min(s,d)} - \phi_{min(s,d)}$ being negative for all s, we see that the first term on the RHS of (A.6.15) is positive. Using (A.6.12) we see that the second term on the RHS is also positive. Hence

$$\sum_s H_{sd} * n_{sd} > 0 \tag{A.6.16}$$

To establish that there is an increase in varieties delivered to d [defined in (A.6.1)], all that remains is to show that H_{sd} equals S_{sd}.

Applying (T1.2b) in (A.6.9) gives

$$H_{kd} = \frac{N_{kd}\Phi^{(\sigma-1)}_{min(k,d)}\delta^{\sigma}_{kd}\left(\frac{W_k T_{kd}}{\beta}\right)^{1-\sigma}}{\sum_r N_{rd}\Phi^{(\sigma-1)}_{min(r,d)}\delta^{\sigma}_{rd}\left(\frac{W_r T_{rd}}{\beta}\right)^{1-\sigma}} \tag{A.6.17}$$

Then via (T1.1b) and (T1.1a)

$$H_{kd} = \frac{N_{kd}\delta^{\sigma}_{kd}\left(P_{\bullet kd}\right)^{1-\sigma}}{\sum_r N_{rd}\delta^{\sigma}_{rd}\left(P_{\bullet rd}\right)^{1-\sigma}} \tag{A.6.18}$$

Via (T1.2a) and (T1.3a) we obtain

$$H_{kd} = N_{kd}\frac{Q_{\bullet kd}}{Q_d} * \frac{P_{\bullet kd}}{P_d} \tag{A.6.19}$$

Finally, via (T1.3b) and (T1.7d) we arrive at

$$H_{kd} = S_{kd} \tag{A.6.20}$$

Increase in the cost of a unit of MM commodity in d

Using (A.6.20), we can write (T1.2a) in percentage change as:

$$\left(1-\sigma\right) * p_d = \sum_s S_{sd} * n_{sd} + (1-\sigma) * \sum_s S_{sd} * p_{\bullet sd} \tag{A.6.21}$$

where

p_d and $p_{\bullet sd}$ are percentage changes in the variables denoted by the corresponding uppercase symbols.

Treating W_s and T_{sd} as constants we use (T1.1a) and (T1.1b) to eliminate $p_{\bullet sd}$ from (T6.21):

$$p_d = \left(\frac{1}{1-\sigma}\right) * \sum_s S_{sd} * n_{sd} + \sum_s S_{sd} * m_d - \beta * \sum_s S_{sd} * \phi_{min(s,d)} \tag{A.6.22}$$

In percentage change form (T1.4a) can be written as:

$$q_{min(s,d)} - \phi_{min(s,d)} = z_{sd} - \frac{M_d}{M_d - 1} * m_d \tag{A.6.23}$$

where

z_{sd} and m_d are percentage changes in the variables denoted by the corresponding uppercase symbols.

Substituting from (A,6.23) into (A,6.15) gives

$$\sum_s S_{sd} * n_{sd} = \frac{1-\sigma}{\sigma}\sum_s S_{sd} * \left(z_{sd} - \frac{M_d}{M_d - 1} * m_d\right) + \left(\frac{1-\sigma}{\sigma}\right)\sum_k S_{kd}\phi_{min(k,d)} \tag{A.6.24}$$

Substituting from (A.6.24) into (A.6.22):

$$p_d = \frac{1}{\sigma} * \left[\sum_s S_{sd} * \left(z_{sd} - \frac{M_d}{M_d - 1} * m_d \right) + \sum_k S_{kd} \phi_{\min(k,d)} \right] + m_d - \beta * \sum_s S_{sd} * \phi_{\min(s,d)} \qquad \text{(A.6.25)}$$

Rearranging:

$$p_d = \frac{1}{\sigma} * \sum_s S_{sd} * z_{sd} + \left(-\frac{1}{\sigma} * \frac{M_d}{M_d - 1} + 1 \right) * m_d + \left(\frac{1}{\sigma} - \beta \right) * \sum_k S_{kd} \phi_{\min(k,d)} \qquad \text{(A.6.26)}$$

With a reduction in competition in d's market, z_{sd} is positive for all s. Consequently, the first term on the RHS of (A.6.26) is positive.

In the second term, the reduction in competition means that m_d is positive, but to sign this term we need to call on (T7.2a) and (T7.2b). From (T7.2a) we find that

$$\frac{M_d}{M_d - 1} = \Gamma_d \qquad \text{(A.6.27)}$$

and from (T7.2b) we find that

$$\frac{\Gamma_d}{\sigma} = \frac{1}{1 + \dfrac{\sigma - 1}{Ntot_d}} \qquad \text{(A.6.28)}$$

Because $\sigma > 1$ and $Ntot_d$ is positive (A.6.28) implies that

$$\frac{\Gamma_d}{\sigma} < 1 \qquad \text{(A.6.29)}$$

Combining (A.6.29) and (A.6.27) we see that the second term on the RHS of (A.6.26) is positive.

Using the definition of β in (A.2.10), we have

$$\beta = \left(\frac{\alpha}{\alpha - (\sigma - 1)} \right)^{1/(\sigma-1)} > 1 \quad . \qquad \text{(A.6.30)}$$

With $\sigma > 1$, $1/\sigma$ is less than 1. Hence

$$\frac{1}{\sigma} - \beta < 0 \qquad \text{(A.6.31)}$$

Combining (A.6.31) with (A.6.12) and (A.6.20), we see that the third term on the RHS of (A.6.26) is positive.

With all three terms on the RHS of (A.6.26) being positive, we conclude that p_d is positive.

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